Two Roles of Debt and Optimal Capital Structure

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Abstract

This paper examines the optimal capital structure of the firm when debt has the roles of both "reducing-free-cash" and "reorganization". Under one reorganization process, we show that the optimal financing policy for the firm is to choose debt seniority structure (between junior and senior debt) as well as to determine the debt level as a whole.

Key Words: debt, optimal capital structure, junior debt, debt seniority, reorganization

1. Introduction

Over the last decade, increasing attention has focused on the positive effects that the debt issue of the firm has on corporate efficiency. Debt plays two roles in increasing the firm value: one is the "reducing-free-cash" role presented by Jensen (1986), and the other is the "reorganization" role following default (see, for example, Wruck (1990)).

Recognizing these benefits of debt, several papers have studied the optimal capital structure of the firm (for example, Stulz (1990), Harris and Raviv (1990), Aghion and Bolton (1992), and Chang (1992)). It can be pointed out, however, that all these studies focus on only one role of debt, that is, either on "reducing-free-cash" or on "reorganization". Therefore, a natural question arises: What is the optimal capital structure for the firm if debt has both roles? The purpose of this paper is to provide one answer to this question.

To attain our aim, we extend the Stulz (1990) model which analyses capital structure focusing on the "reducing-free-cash" role of debt. Introducing the "reorganization" benefit of debt into Stulz model, we examine the optimal financing decision of the firm when debt

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plays two roles. Under one reorganization process we assume, we show that the issue of junior debt can increase the firm value by exploiting two kinds of benefits of leverage more effectively, and that the optimal financing policy for the firm is one which chooses the debt seniority structure (between junior and senior debt) as well as determining the debt level as a whole.

2. Two Kinds of Benefits of Debt and Capital Structure

We will begin by presenting the Stulz (1990) model as a starting point of our analysis. The firm's assets in place at date 0 yield a random cash flow $R$ at date 1 which follows a probability density function $g(R)$. Investment opportunities are available to the firm at date 1. $Z (>1)$ and $Y (<1)$ are the expected returns of good (positive NPV) and bad (negative NPV) projects, respectively. The firm can invest in good projects only for the first $I^*$ units invested, thereafter it has only bad projects to invest in. Hence, $I^*$ is the optimal amount of investment for the firm. However, as managers derive perquisites from investment, it is not assured that they realize $I^*$ under information asymmetries. The firm issues debt with a face value of $F$ at date 0 to be repaid at date 1. Then, the value of the firm at date 0 with debt issue, $V(F)$, is given by equation 4 in Stulz (1990, p.14). We rewrite this as follows:

$$V(F) = V^* - D(F)$$  \hspace{1cm} (1)

where

$$V^* = E(R) + (Z - 1)I^*$$  \hspace{1cm} (2)

$$D(F) = \int_{F+I^*}^{\infty} (1 - Y)((R - F) - I^*) g(R) dR$$

$$+ \int_{F}^{F+I^*} (Z - 1)(R - F) g(R) dR + \int_{0}^{F} (Z - 1)I^* g(R) dR.$$  \hspace{1cm} (3)

Equation (1) states that the value of the firm $V(F)$ is the difference of the maximum firm value, $V^*$, which equals to the expected cash flow $E(R)$ plus the NPV of the good projects $(Z - 1)I^*$ (See(2)) and $D(F)$ which expresses the agency costs which result from the "free-cash-flow problem". The first term in (3) corresponds to the overinvestment cost arising from free cash flow $((R - F) - I^*)$, whereas the second and third terms imply underinvestment cost caused by managers' lack of credibility when the cash flow is truly low.\footnote{1}

From (1), optimal amounts of debt issue $F^+$ must satisfy\footnote{2}

$$D^*(F^+) = 0.$$  \hspace{1cm} (4)

It is easily seen that this condition exactly coincides with the optimality condition in the
Stulz model (equation 5 in his paper). Hence, we can say that the optimal amount of debt issue in the Stulz model is the one which minimizes the agency costs of the free-cash-flow problem $D(F)$.

While Stulz analyzed the "reducing-free-cash" role of debt, we should also notice another benefit of debt, "reorganization", that is, debt issue raises the value of the firm by generating the possibility of reorganization by debt holders following default. We will introduce this benefit into the Stulz model, and re-examine the optimal capital structure of the firm.\(^3\)

The Stulz model assumes that if the firm defaults ($R < F$), the firm is liquidated and the investment opportunities of the firm disappear. In contrast, we suppose that in the event of a default, the firm's control rights are transferred from incumbent managers to debt holders, and debt holders carry out a reorganization; debt holders then investigate $R$, supply funds $I^* - R$ if $I^* > R$ (receive $R - I^*$ if $R > I^*$), and realize the efficient investment level $I^*$. To achieve this reorganization, debt holders incur costs $C(F)$. We assume $C'(F) > 0$ because as the firm issues more debt, the number of debt holders increases, so coordination or collective action costs rise more.

To make our analyses as simple as possible, we also assume that the NPV of good investment is large relative to reorganization costs ($(Z - 1)I^* > C(F)$), so that debt holders always have an incentive to carry out a reorganization following default.

Then, we can express the expected value of a reorganization, $M(F)$ as

$$M(F) = \int_0^F [(Z - 1)I^* - C(F)] g(R) dR. \quad (5)$$

Thus, the value of the firm with reorganization, $V_1(F)$, can be obtained by adding $M(F)$ to $V(F)$. That is,

$$V_1(F) = V^* - D(F) + M(F). \quad (6)$$

From (6), we obtain the optimal debt level with two benefits of leverage. The optimal amount of debt issue, $F^*$, must satisfy

$$D^*(F^*) + M'(F^*) = 0. \quad (7)$$

Equation (7) means that the optimal level of debt is determined by the marginal effects on both agency costs of free-cash-flow problem $D(F)$ and reorganization benefits $M(F)$. This is a natural result from our supposition that leverage has two kinds of benefits, that is, "reducing free-cash" and "reorganization".

However, it is important to note that when debt has these two roles, the financing policy that adjusts only the debt level, $F$, cannot fully exploit the two benefits of leverage. This is because there may be a trade-off between minimizing agency costs, $D(F)$, and max-
imizing reorganization benefits, $M(F)$. For example, suppose that $I^*$ is relatively low. From the view of minimizing $D(F)$, it is desirable for the firm to raise $F$ in order to reduce (the expected) excess cash flow. On the other hand, from the view of maximizing $M(F)$, it is profitable to lower $F$ when $I^*$ is small, since NPV of good investment gained by reorganization, $(Z - 1)I^*$, is low. That is, when debt has two kinds of roles, a conflict arises in determining $F^*$.

The point we wish to emphasize is that there may exist a better financing device for the firm to solve this problem. This is one in which the firm issues junior debt and determines its level as well as the level of debt as a whole. Then, the two benefits of leverage can be realized more efficiently. We will show this in the next section.

3. Junior Debt Issue and Optimal Financing Policy

Consider a firm which issues two types of debt with different seniority at date 0; one is junior debt whose face value is to be repaid at date 1 is $F_J$, and the other is senior debt which has a face value $F_S$. As before, the total amounts of debt issued, $F$, equals the sum of two types of debt ($F = F_J + F_S$).

When $R < F$ at date 1, the firm falls into default. However, the reorganizing process following default is different to the previous section. In this case, since the firm has two kinds of debt, we can think of several ways for how a reorganization might be carried out. In the following analysis, however, we assume one particular reorganizing process, and examine the optimal financing policies of the firm under it.

We separate the event of default ($R < F$) into two cases. Case 1 is when $F_S \leq R (< F)$, and Case 2 is when $R < F_S (< F)$.

In Case 1, the firm has enough cash flow ($R$) to repay senior debt ($F_S$) at date 1. Then, according to the absolute priority rule, senior debt holders can receive full repayment $F_S$. Hence, control rights of the firm are transferred to junior debt holders whose claim cannot be fully repaid. Consequently, only junior debtors reorganize the firm in case 1. We express the costs of reorganization as $C(F_J)$, the same functional form as $C(F)$ and also assume reorganization is always valuable for junior debtors, that is, $(Z - 1)I^* > C(F_J)$.

In Case 2, the firm can repay neither senior debt nor junior debt. Then, control rights of the firm are transferred to both debtors. However, it seems difficult for two different classes of debtors to take collective action for reorganization, since there arises serious conflicts of interests between the two parties in making several decisions on reorganization process (especially with respect to the obligations and returns of each party). Hence, for simplicity, we assume that in case 2, debt holders cannot reorganize the firm because the costs of collective action are too high.

Thus, under the reorganization process assumed, reorganization occurs only when $F_S$
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\( R < F \) (in case 1), and it is carried out by junior debt holders. Therefore, the expected value of reorganization can be expressed as

\[
M_F(F, F_J) = \int_{F-F_J}^{F} \left[ (Z-1)I^* - C(F_J) \right] g(R) dR
\]  

(8)

noting that \( F - F_J = F_S \). Then, we can write the value of the firm \( V_2(F, F_J) \) as

\[
V_2(F, F_J) = V^* - D(F) + M_F(F, F_J).
\]  

(9)

Therefore, the firm’s optimization problem can be described as

\[
\max V_2(F, F_J) \quad \text{subject to } F_J \leq F.
\]  

(10)

Let \( F^{**} \) and \( F_J^{**} \) denote the solutions of (10), that is, the optimal amounts of total and junior debt, respectively.

Now, we can compare two financing policies of the firm when debt has two roles. One is to choose only the debt level \( F \) (eq.(6)) which we analyzed in section 2, and the other is to determine the junior debt level, \( F_J \), as well as \( F \) (eq.(10)). The following result shows that the latter dominates the former.

**Proposition 1.** If debt has both “reducing-free-cash” and “reorganization” roles, a financing policy with the issue of junior debt can realize a higher firm value than a policy which chooses only the debt level. That is, \( V_2(F^{**}, F_J^{**}) \geq V_1(F^*) \).

**Proof.** In (8) and (9), note that if \( F_J = F \), \( M_F(F, F_J) \) coincides with \( M(F) \), hence \( V_2(F, F_J) = V_1(F) \). Thus, we know that maximizing \( V_2(F, F_J) \) with the constraint \( F_J = F \) is exactly the same as the problem of maximizing \( V_1(F) \). Therefore, in the maximization problem (10), if we obtain \( F_J^{**} = F^{**} \) (a corner solution), then \( V_2(F^{**}, F_J^{**}) = V_1(F^*) \), and if we obtain \( F_J^{**} < F^{**} \) (an interior solution), then \( V_2(F^{**}, F_J^{**}) > V_1(F^*) \). Q. E. D.

Proposition 1 suggests that when debt has two roles, the optimal financing policy for the firm is to issue junior debt and choose its optimal level as well as the total debt level. This implies that the debt seniority structure (the choice between junior and senior debt) can be an important matter in the firm’s financing decision. An intuitive explanation for Proposition 1 may be that by issuing junior debt, \( F_J \), and making it play a role in “reorganization”, the level of debt \( F \) can be determined so that debt as a whole can reduce the agency costs of free-cash-flow problem \( D(F) \) more effectively. Hence, the two benefits of debt can be exploited more. The next proposition shows this intuition more clearly under the uniform distribution assumption.
Proposition 2. Suppose that $R$ is uniformly distributed. That, if we obtain a interior solution in (10), that is, $F^* < F^{**}$, the optimal debt level, $F^{**}$, minimizes the agency costs of free-cash-flow problem, $D(F)$, and the optimal junior debt level, $F_{j}^{**}$, maximizes the expected value of reorganization, $M_{j}(F, F_{j})$.

Proof. When $R$ is uniform on $[0, R_U]$, then $g(R) = 1/(2R_U)$. Under this assumption, the expected value of reorganization, $M_{j}(F, F_{j})$, can be expressed as a function of $F_{j}$:

$$M_{j}(F, F_{j}) = \frac{1}{2R_U} \left[(Z-1)I^{*} - C(F_{j})\right] F_{j} = M_{j}(F_{j}).$$

(11)

That is, it does not depend on $F$. Then, the first order conditions for an interior optimum to (10) are

$$- D^{'}(F^{**}) = 0, \quad M_{j}^{'}(F_{j}^{**}) = 0,$$

(12)

which means that $F^{**}$ minimizes $D(F)^{'}$ and $F_{j}^{**}$ maximizes $M_{j}(F, F_{j}) = M_{j}(F)$. Q. E. D.

Proposition 2 implies that when $R$ follows an uniform distribution, two benefits of leverage can be fully exploited by a financing policy with junior debt issue, in the sense that both the minimal value of $D(F)$ and the maximal value of $M_{j}(F, F_{j})$ are realized. While debt as a whole plays a role in "reducing-free-cash", junior debt maximizes the benefits of "reorganization". That is, each financing variable has a role. Due to this separation, the two roles of debt function most effectively.

4. Concluding Remarks

In this paper, we analyzed the optimal capital structure when debt has both the roles of "reducing-free-cash" and "reorganization". Under a particular assumption about the reorganization process, we have shown that the optimal financing policy for the firm is to choose debt seniority structure (between junior and senior) as well as the total debt level. We conjecture this result can also be obtained under a more general reorganization process than the one assumed. We leave this extension to future research. The main goal of this paper is to present the possibility that debt seniority structure matters in the firm’s financing decision when debt has two kinds of benefits.

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References

1 For details of how the costs of over- and under investment arise, see Stulz (1990).
2 Throughout this paper, we assume that the second order condition for optimality is always satisfied.
3 We consider the firm's capital structure as the determination of the relative amounts of each securities, taking the types of securities as exogenous. That is, the study on securities design lies outside the scope of this paper.
4 We see this by noticing that \(dF^{++} / dl^* < 0\) holds from (4).
5 Let \(F^{++}\) denote the debt level which maximizes \(M(F)\), then we obtain \(dF^{++} / dl^* > 0\) from \(M' (F^{++}) = 0\).
6 Stulz (1990) also points out that debt issues with different seniority may be useful but his logic is very different from ours. He shows that in the presence of *agency costs of debt*, strip-financing and private debt issues with different seniority can increase the firm value.
7 Also note that \(F^{**} = F^+\)