On the Optimal Allocation of Three Modes for The Intercontinental Transportation of Seasonal Products

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【Abstract】 The existence of a worldwide trend that the world’s manufacturing site is moving to East Asia is well known. Very large part of the products produced in these districts are to be transported to the U.S.A. and are concentrated to rather small number of ports, including airports. These situations are causing very serious traffic problems. New products are required to be transported swiftly by air. Once the market demands are stable, the products should be sent rather slowly but in larger amount. Since the airport of China is quite restricted, transportation cost is increasing and also the time for transportation is increasing. Here, the third method is appearing. This is the so-called Sea-and-Air transportation. Its cost and time in transit takes a mean position between Air and Ocean freight services. However, there have been existing no reasonable strategy how to allocate these three methods for the transportation. In the previous paper the optimal condition was considered under the assumptions that products are transported directly after their production. Here, the same problem is considered under the assumption that all products are already produced and stored in the inventory. Products are transported following the demands from the consuming districts. The most efficient transportation for such case is considered with the help of simulations.

1 Introduction

As is shown in the previous paper [3], it is a well known tendency that many advanced nations have established many firms in the far east for the cheap labor cost and transported their products to many world consuming countries. These consuming countries are usually developed countries and the production sites are rather restricted. Therefore, the products are concentrated to rather small number of hub ports for their transportation. As these products, there appear many seasonal products such as textile products or Xmas gifts. Every year such kinds of product appear seasonally and their movements resume similar styles. Although some products are not seasonal, such as cellular phone or some electronics devises, manufacturers produce new models rather cyclically and as a result their movements assume the same styles. We may consider that the cycle is usually 6-months long from experiences, although the cycle does not have special meanings.

Generally the transportations of these products have particular characteristics. The new products should be sent as rapidly as possible after their production, because, the early appearance gives the new products large competitive advantages. For that reason the air transportation is usually used at this stage. However, since there are more products to be sent by air, air transportation is becoming more difficult in both economically and in terms of volume of transportation. The location of production is rather concentrated in small area, and as a result, the airport to be used is also restricted.

After the first period of such quick transportation, a slower but large amount of transportation is needed. For this transportation, the one by sea is the usual selection. Although the transportation by sea is slow, the cost of the transportation is very low and the amount to be sent at one time is extremely large. The difference of these two methods is extensively large both from the point of cost and from the point of the capacity. It is natural that the third method of medium speed and medium amount become necessary. This method is the so called sea-and-air transportation. Generally, this method means a combinational transportation.

At first the products are sent by sea to some neighboring airport where the services are obtainable more easily and then they are sent by air as further transportation to the final destination. The most popular routes are the ones through Japan. The details of this situation are shown in the paper [4]. The cost for this new method is almost half and the time required is almost half of direct sea.

Introducing this method, however, casts a difficult problem of how much this method costs and moreover how much should be sent by this method. On this problem, only few works have been appeared [1] [2]. The calculation of the optimal allocation of these methods is first considered in this series of research. In the previous paper [3], only the basic
characteristic of the problem is considered. In this paper optimal allocation is considered on the more realistic assumption on the speed of the consumption. This report is consisted as follows. In 2, the importance of this method in relation to Japan is specially presented. Section 3 is devoted to the several assumptions for the description of the system. In 4 the costs and obtainable values by three modes are considered and from them the final profit is calculated. Algorithms for the calculation of the profit is shown in 5. The efficiency of the introduction of sea and air mode is considered in 6. The concluding remark is given in 7.

2 Sea-and-Air Transportation for Japan

The third transportation mode of sea-and-air has special meanings to Japan. In this section the meanings of this mode in relations to Japan is presented. Because, this method also proposes an important economical problem of how to develop the infrastructures of airport in Japan.

First of all, it should be borne in mind that the largest consuming district of the above mentioned product produced in the far east of Asia is the U.S.A. Moreover, among the production sites of east of Asia, the Shanghai district is the largest. From the situation, it is natural that Shanghai Airport is the most reasonable airport to be used for the air transportation of such products. The amount of such products intending to use Shanghai airport is tremendously large. The situation makes the problem even more complicated.

On the other hand, because of the sluggish economy in Japan, obtaining air services is not difficult. The amount of the transportation to the U.S.A. is tremendously large and it is not an exaggeration to say that it constitutes the world transportation circumstances. Therefore, among the third method of transportations mentioned in the previous section, the one through Japan is most important. Therefore, in calling the third mode of sea and air transportation, the one through Japan is typical and here, when calling sea and air, we mean the one with the route through Japan.

In this mode called sea-and-air transportation the products are once sent to Japan by sea and then transported by air to the U.S.A. The time required for this mode is about a week to ten days and the cost for this is about half of the direct air transportation from China to the U.S.A. However, this method is not applied regularly, and the application of this is quite optional and not in efficient way. The low cost for this is justifiable in many ways. If this method can be proven theoretically to be efficient, the usage of this mode would be more systematically performed. This would be profitable for Japan. This is also the motivation of this report. One most important of the reason is that the ship used from China is the returning one used for transportation from Japan to China. Moreover the air cost from Japan to the U.S.A. is comparatively low. Under these conditions, we may say that this mode of
transportation is fairly promising. These situations are to be explained in the accompanying paper [4].

However, the transportation strategy for using optimally these methods has not been developed until today. This is also the motivation of this report. It is expected that as the result of this new proposal, a direction of the improving infrastructure of the traffic systems of Japanese ports or airports can be derived.

3 System Description

In this section several characteristics of the system is presented first and then by making some assumptions the way to find the optimal transportation is studied.

3.1 Characteristics of The System

It should be noted that these seasonal products should be transported to the consuming district as fast as possible once they are produced. They are new products and the swift appearance gives the products great competitive advantages. After the first period, the products are sent in large quantities to respond to the steady demands of the consuming districts and therefore the value of them decreases. This is the first characteristics. To start the analysis the time variable \( t \) should be defined. It represents the time in the destination.

**definition 1** (Definition of Time) In this report time \( t \) start when the first product arrives at the destination.

![Figure 1: Schematic View of Value Function \( f_v \)](image-url)
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(\textit{Assumption I}) The value of the product monotonically decreases as time elapses. Then it approaches to a fixed value. How rapidly the value decays and how high the first value is varies much depending on the characteristics of the products.

This is the basic assumption and there can be many variations depending on the products. The most important of them is the situation of the consuming amount. It may change dynamically. For the advanced analysis, these situations should be considered.

At the same time, the swift transportation costs much and this is the second characteristic. Here the following assumption is introduced. (\textit{Assumption II}) The transportation cost decreases monotonically as the required time increases. The cost by air is about hundred times higher than those by sea. And the cost by sea-and-air is about half to one-third of the cost by air.

On the top of the cost of transportation, the cost for the inventory should be considered. (\textit{Assumption III}) Inventory cost is a linear function of length of storage time and the amount of products.

3.2 Stand Point of the Analysis

Here the basic assumption for the analysis is presented. The total system of the inter-modal transportation involves many enterprises, consisting a very complicated system. Usually production, transportation and retailing are conducted by different companies and therefore their advantages and disadvantages are conflicting each other. These situations make the analysis considerably complex. In the analysis a different stand point may leads to a quite different results.

In this report, our stand point is that of retailers one. And it is first assumed that all the products are already stored in the sender’s inventory. However, the transportation should be conducted by the usual ways. The problem, therefore, is as follows.

If the amount of consumption and speed of it is assumed first, then what mode of transportation and when should it be performed to satisfy their consumption?

3.3 Modeling

For the analysis of this system, several parameters and functions are needed to be given. For the sake of simplicity, the transportation by air, by sea-and-air and by sea are called \(A\)-mode, \(SA\)-mode and \(S\)-mode, respectively. Here, the following parameters are introduced.
3.3.1 parameters

$s$: Amount of the product sent by one time of transportation by $A$-mode.

$d$: Unit time span. Basically this is day.

$\mu s$: Amount of product sent by one time transportation by $SA$-mode.

$\nu s$: Amount of product sent by one time transportation by $S$-mode.

$T$: Total inventory time. That means this is the time from the beginning until there exists no product in the inventory.

(note) The above parameters indicates that $\mu$ times of $s$ are sent at one time by $SA$-mode and $\nu$ times amount are sent by $S$-mode. These situations are caused by adopting containers for the transportation by sea and not for air. For $SA$-mode, CFS (container freight station) cargo are used.

3.3.2 functions

On the basis of these assumptions several nonlinear functions are needed for the mathematical analysis of the problem. First of all, as is indicated in the previous subsection, estimated total consuming amount and the estimated consuming speed are to be given first. The estimated total consuming function $GC(t)$ is a function of $t$, indicating the amount of the total consumption of the products until $t$ from the beginning. This function is defined as

$$GC(t) = 1 - \exp(-\omega t)(\omega t + 1)$$  \hspace{0.5cm} (1)

Figure 4 shows the schematic view of this function. This figure indicates a model of how the consumption progresses. The consumption begins from 0% at $t = 0$ and monotonically increases until 1, that is 100%, after about 6 months or so.

Although it is an idealistic model, it is usual that the retailers estimate the progress of their sales and make the plan of transportation of the product. On this curve, more precise explanations are given in 3.3.4.

The curve $f(t)$ or transportation cost curve $f_{tc}(t)$ are also needed to be assumed. They are defined as follows.

$f(t)$: The value of one unit product produced at $t = 0$ that means the transportation is started.

$f_{tc}(t)$: The cost of transportation of one unit product transported after $t$ of its transportation starts.

$f_{tc}(t)$: The cost of inventory of one unit for $t$ after its storage begins.
Owing to the assumption of monotone decreasing property, we may choose exponential functions of time such as

\[ f_v(t) = a \exp(-\alpha t) + e, \]  

as a candidate of the value curve, where \( a, \alpha, e \) are constant, decided by the characteristics of the product. In the case of the new models of goods, \( a \) is usually extremely high and decreases rapidly. That means \( \alpha \) is rather large. This is also the case of products in severer competitive conditions. The speedy introduction to the market is strongly required. The value decreases monotonically and approaches to certain fixed value \( e \). A candidate of the function for the transportation cost is given as

\[ f_{tc}(t) = b \exp(-\beta t) + h, \]  

where \( b, \beta, h \) are also constant, decided by the characteristics of the product. It is also determined by the size and other characteristics of the products. Of course, this curve does not mean there exists every service on this curve. It should be noted that this \( f_{tc}(t) \) indicates a cost of one time service to transport unit product at \( t \). It should be noted that there exist only three services on this curve, that is \( A \)-mode, \( SA \)-mode and \( S \)-mode. In the usual case, the cost by air is about 100 times larger than that of by sea and the time required for this transportation is 1 days by air and 10 days by sea and air and 30 days by sea. That means \( h \) is comparatively small and almost 1/100 of \( b \) although it may not be zero. This situation implies that the cost curve approaches very near the static value \( h \) after almost 30 days from the system started. Here for the future use, the following time interval constants are introduced.
definition 2 Time constants $t_a$, $t_{sa}$ and $t_s$ are defined as:
$t_a$ is a required time for one time of $A$-mode transportation.
$t_{sa}$ is a required time for one time of $SA$-mode transportation.
$t_s$ is a required time for one time of $S$-mode transportation.

Schematic views of these value function and cost function are given in Figure 1 and 2.
From the obtained values, the inventory cost should also be subtracted. It is natural that
the inventory cost is a linear function of time. Therefore it is given as

$$f_{ic}(t) = \gamma t.$$ (4)

Note that this is the one for a unit product.

3.3.3 Assumption on the Modeling

In the previous report the most basic situation for the transportation is considered. That means
the products are basically transported directly after the production without saving in the
inventory. More precisely, products are saved in the inventory only to wait until the proper
amount for one of the three mode transportation is available. However, this does not
necessarily reflects the actual circumstances. At present, production and transportation are
conducted by different enterprises and therefore, all the products should be sent to the
destination districts must be delivered at one time.

There comes the new problem. In the consuming district there exists constant consuming
demands and the total consuming amount is simply exhibited by the gross consuming
function.

(Assumption IV) All the products to be sent are stored in the inventory at the beginning. The analysis starts when the first product arrives at the destination.

(Assumption V) The total consuming amount is a monotone increasing function of time. The total transported amount should not be allowed to come under this function at any time.

![Schematic View of Total Consuming Function GC(t)](image)

The most basic transportation system was analyzed in the previous report. In the report the transportation are performed soon after the production. In this report the production have already finished before the transportation. However, these both cases do not fully reflects the actual situation. The actual cases are more complicated. These cases are to be analyzed in the future works.

### 3.3.4 Estimated Consuming Amount and Total Transportation Amount

Here the following two functions should be introduced. One is the function describing the total consuming function as the function of $t$ and total transportation amount function. The total consuming function indicates the overall consuming amount from the beginning. The consumption begins from the start and increases and then comes to zero. The total consuming function is the integral of this amount of consumption and therefore it is the monotone increasing function and approaches to a certain fixed value. Here it is denoted as 100%.

As a candidate of this function, the function given by (1) is considered. Figure 4 shows the schematic view of this function. Three curves represent cases of $\omega = 0.04$, $\omega = 0.035$, $\omega = 0.03$ from above. This function is known as the function showing the response of a 2nd order control system to a unit step input function. It denotes how the total consumption reaches to
100% of the supplied products after the start of the consumption. \( \omega \) is a parameter showing the speed of the response. In case \( \omega \) is large, the response becomes fast. Figure 6 shows the local view of this function around the start point.

\[
G(s) = \frac{\omega^2}{(s + \omega)^2}
\]  

Figure 5: Transfer Function for Consumption

Figure 6: Schematic View of Total Consuming Function \( GC(t) \) (Local View)

This curve is known as the initial response of a transfer function of so-called 2nd order delayed system given as

where \( s \) is Laplace variable. It is conceived on the consideration that the consumption may pick up speed after some initial hesitation. The consuming speed in the destination district is shown as Figure 8. If the total consumption is given as (5), then the speed of the consumption
is given as

\[ SGC(t) = \omega^2 te^{-\omega t} \]  \hspace{1cm} (6)

These curves are crude estimations of the actual consumptions. In the applications of the theory, they should be constructed on the more precise and realistic analysis. The supply of the products should be accomplished assuring these consumption. In Fig.9 the step functions indicate the transported amounts. The first part was done by air and the second part is

![Figure 7: Schematic View of Total Consuming Function GC(t) (Very Local)](image)

![Figure 8: Schematic View of Consuming Speed](image)
accomplished by sea-and-air and then the third part is done by sea. The amount of the transportation of the three modes are quite different and the amount transported by air is about 1/100 of the amount by $S$-mode and can not be shown here. It is difficult to show the supply and consuming relation in one figure. The shadowed area in the figure show that products are in the inventories and cost for that. As is easily seen from the figure, since the transportation ability of sea is fairly large, the inventory time is comparatively large. Although the transportation cost for $S$-mode is very low, the inventory cost becomes large.

3.3.5 Calculation of the Obtained Value

The speed of the consumption means the base of the obtainable values of the product. The value of the product varies according to time. Therefore, the product of value and speed of the consumption implies the obtained value per day. Here the following function indicates this.

$$V_{obt}(t) = SGC(t) \cdot f_v$$
$$= (\omega^2 t \exp(-\omega t))(a \exp(-\alpha t) + e)$$  \hspace{1cm} (7)
Fig. 10: Schematic View of Obtained Value

Fig. 10 shows the schematic view of this function. Total value obtained by this product increases monotonically as \( t \) and it can be interpreted as the area made by the above function and horizontal axis. This value is a fixed value. It is the promised value from the beginning. This figure shows that the obtainable values are very high for the early time of the transportation. That means the swift transportation is fairly efficient for the sales. Naturally, the swift transportation costs much. As was shown in the previous paper for one product

\[
\int_0^\infty V_{obt}(t) \, dt \quad (8)
\]

shows the obtainable profit and the time point giving the peak of the curve is the best point for the transportation. Of course there exist only three services of the transportation. It can be safely said that the nearest one becomes the most efficient service. The final profit is obtained by subtracting the transportation cost from these obtainable values. Mathematically, the total obtained value can be calculated as the integral of the area between the obtainable value curve \( V_{obt}(t) \) and the x-axis, that is

\[
\text{Total Value} = TV = \int_0^\infty V_{obt}(t) \, dt \quad (9)
\]

In this report it is assumed that this value is fixed.
3.3.6 Calculation of the Cost

To assure the above calculated total value two kinds of cost should be subtracted. They are:
1. Transportation Cost
2. Inventory Cost

![Figure 11: Value -Transportation Cost](image)

To mathematically define the problem, variables $x$, $y$ and $z$ are to be defined.

**Definition 3**
- $x$ denotes the number of transportation by $A$-mode.
- $y$ denotes the number of transportation by $SA$-mode.
- $z$ denotes the number of transportation by $S$-mode.

The transportation cost by $A$-mode, $SA$-mode and $S$-mode are denoted as $TC_A(x)$, $TC_{SA}(y)$ and $TC_S(z)$, respectively. That means they are nonlinear functions of $x$, $y$ and $z$ respectively. The total cost $TC$ is given as

$$TC = TC_A(x) + TC_{SA}(y) + TC_S(z)$$

(10)

As for the inventory cost, it is a linear function of both number of stored products and the length of the stored time.
\[ IC = (\text{number of stored products}) \times (\text{stored time}) \] (11)

Therefore, the inventory cost is shown as the area defined by the vertical line of start of storage, consuming curve and the transported amount. In Figure 9 these area are shown by shadowed area. Since the consuming curve are fixed, the transportation should be decided to reduce this cost.

3.3.7 **Constraint on the Transportation**

The total consuming curve and consuming speed of the product are fixed, and some parts of the transportation are to be automatically decided. Since the time required for the \(SA\)-mode and \(S\)-mode are at least 8 days and 20 days more than \(A\)-mode, the products which are consumed during the first 8 days should be transported by \(A\)-mode and those consumed during the first 20 days should be transported by \(A\)-mode or \(SA\)-mode. In the example shown by Figure 7, before 8 days a little less than 2 \%, for slower case, around 3 \% for faster case of the product should be transported by \(A\)-mode and about 10\% for slower case about 15\% for faster case should be transported by \(A\)-mode or \(SA\)-mode.

3.4 **Problems Description**

To start the problem, the following restriction must be introduced.

- (H1) Value curve and cost curve are fixed.
- (H2) Inventory cost is a linear function of time.
- (H3) The transportation should be restricted to three method.
- (H4) Each transportation method has fixed transportation amount and until the amount of the products satisfy this restriction, the transportation should not be performed, except the last one by sea.
- (H5) The transportation amount of sea-and-air is \(\mu\)-times larger that that of air.
- (H6) The transportation amount of sea is again \(\nu\)-times larger that that of air.
- (H7) Sea transportation is not zero for all cases.

**Problem** Whether the \(SA\)-mode is efficient to introduce. And if so, how much amount should be allocated to \(A\)-mode \(SA\)-mode and \(S\)-mode and what are the point \(t = \tau_{SA}\) and \(t = \tau_{S}\) which make the profit maximum, where

- \(t = \tau_{SA}\) is the time at which the storage by \(SA\)-mode begins
- \(t = \tau_{S}\) is the time at which the storage by \(S\)-mode begins
This indicates that the transportation by air is performed from $t = 0$ to $t = \tau_{SA}$. Then from $t = \tau_{SA}$ to $t = \tau_S$ the transportation by sea-and-air and after $t = \tau_S$, the transportation is only by sea. Note that these time points are those in the destination, therefore to ensure these points, some modes of transportation must begins at the same time in the sending district. Then, mathematically the problem can be shown as:

**Mathematical description of the problem**  Let the total profit obtained as a function of $x$, $y$ and $z$ by the transportation using three modes be denoted as $TP_{all}(x, y, z)$. Then the problem should be as follows. Find $x, y, z$ which satisfy

$$\text{Max} TP_{all}(x, y, z)$$

where $TP_{all}$ is the total profit function. Although this is the final object, here in this paper, the first part of the problem is only considered, because the problem becomes fairly complicated and it is out of the range of this paper.

4 Calculation of the Profit

Here, the total expected profit by the allocation of the three modes transportation is considered. It is assumed that $x$ times $A$-mode transportation and $y$ times $SA$-mode transportation and $z$ times $S$-mode transportation are performed. The inventory cost and transportation cost are same for each time. However, the value function changes as time elapses.

As is easily understood from the given figure, the product transported by $SA$-mode arrives at the destination after $\tau_{SA}$ and the product transported by $S$-mode arrives at the destination after $\tau_S$. Therefore, before $\tau_{SA}$ only $A$-mode should be adopted and before $\tau_S$ only $A$-mode or $SA$-mode can be used. The problem therefore is that after $\tau_A = 0$ and before $\tau_{SA}$, which is more efficient? $A$-mode or $SA$-mode? This is decided by the following calculation.

$$TP_{all}(x, y, z) = TV - CS - CSA - CS$$

4.1 Cost by the $A$-mode Transportation

By assumption H5, the transportation amount is same at each time and is equal to $a$. Let time points $t_{A1}, t_{A2}, \ldots, t_{Ak}$ be defined as

$$GC(t_{Ak}) = ks.$$  \hspace{1cm} (13)

They are defined as the time at which the total consumption is equal to the total transported
volume. Since the total consuming function is a nonlinear function, these points do not make fixed intervals. As is indicated, $C_A$, the cost of $A$-mode include transportation cost $TCA$ and inventory cost $ICA$, therefore

\[ C_A = TC_A + IC_A \]  

(14)

and they are calculated as

\[ TC_A = xsf(t_a) \]

\[ IC_A = \int_0^{T_A1} (a - GC(t)) \gamma dt + \int_{T_A1}^{T_A2} (2a - GC(t)) \gamma dt \cdots \]

\[ + \int_{T_Ax}^{T_Ay} (xa - GC(t)) \gamma dt \]

4.2 Cost by $SA$-mode Transportation

As in the previous sub-subsection, time points $t_{SA1}$, $t_{SA2}$, $\cdots$, $t_{SAy}$ be defined as

\[ GC(t_{SAk}) = xs + k\mu_s. \]

(15)

Similarly to the previous subsection, the cost by $SA$-mode, $C_{SA}$, included in the total transportation can be calculated as

\[ C_{SA} = TC_{SA} + IC_{SA} \]

(16)

and they are calculated as

\[ TC_{SA} = \mu xsf(t_{SA}) \]

\[ IC_{SA} = \int_{T_Ax}^{T_{SA1}} (xs + x\mu_s - GC(t)) \gamma dt + \int_{T_{SA1}}^{T_{SA2}} (xs + 2\mu_s - GC(t)) \gamma dt \cdots \]

\[ + \int_{T_{SAy-1}}^{T_{SAy}} (xs + y\mu_s - GC(t)) \gamma dt \]

(17)
4.3 Cost by the S-mode Transportation

After A-mode and SA-mode transportation, S-mode transportation is performed. Following the previous sub-subsection, define time point $t_{S1}$, $t_{S2}$, $\ldots$, $t_{Sz}$ as

$$GC(t_{Sk}) = xS + y\mu S + k\nu S.$$  \hspace{1cm} (18)

Following the previous subsection, the total cost of the S-mode, $C_S$, is given as

$$C_S = TC_S + IC_S.$$  \hspace{1cm} (19)

And they are calculated as

$$TC_S = \nu zsf_c(t_s)$$

$$IC_S = \int_{t_{SAy}}^{t_{S1}} (xS + \mu S - GC(t)) \gamma dt + \int_{t_{S1}}^{t_{S2}} (xS + \mu S - GC(t)) \gamma dt \ldots$$

$$+ \int_{t_{Sz-1}}^{T} (xS + \mu S - GC(t)) \gamma dt$$

4.4 Total Profit of the Product

The total profit $TP_{all}$ is obtained by substituting (9), (14), (16) and (19) into (12).

5 Algorithm for the Optimal Transportation

Although there exist three modes of transportation, they are restricted by their required time. Before $\tau_{SA}$, the only available transportation is A-mode. And before $\tau_{S}$, S-mode cannot be applied. The problem to be decided is therefore, after $\tau_{SA}$ whether transportation by A-mode is more efficient than SA-mode or not. This is decided by the comparison between transportation cost and inventory cost. Naturally, since inventory cost is usually much lower than the transportation cost, the consideration on it should be started after mandatory A-mode, SA-mode. As for the efficiency of introduction of SA-mode, it is no need to mention here. It is clear from the figure.

It is same for the start of S-mode. Soon after the minimum SA-mode finishes, S-mode should begin. However, each mode of transportation has fixed amount of transportation. Therefore, only after the consumption reached the already transited amount, the new mode
can be started.

The following is the algorithm.

**Step 1** Divide the required consuming amount at \( t = t_{sa} \) by \( s \). Then \( x \) is given as

\[
x = \left\lfloor \frac{GC(t_{sa})}{s} \right\rfloor + 1
\]

where \( \lfloor \alpha \rfloor \) indicates largest integer which does not exceed \( \alpha \).

**Step 2** \( t_{A1}, t_{A2}, \ldots, t_{Ax} \) are calculated by (13). Each transportation by \( A \)-mode are accomplished such that at these time the amount of inventory increases by \( a \). The time \( t = \tau_{sa} \) is defined as \( \tau_{sa} = t_{Ax} \).

**Step 3** \( y \) is defined by dividing the required transportation amount for \( SA \)-mode by \( \mu s \) as follows.

\[
y = \left\lfloor \frac{GC(t_s) - GC(t_{Ax})}{\mu s} \right\rfloor + 1
\]

**Step 4** \( t_{SA1}, t_{SA2}, \ldots, t_{SAY} \) are calculated by (15). Each transportation by \( SA \)-mode are accomplished such that at these times the amount of inventory increases by \( \mu a \). The time \( t = \tau_S \) is defined as \( \tau_S = t_{SAY} \).

**Step 5** \( z \) is defined by dividing the required transportation amount for \( SA \)-mode by \( \nu s \) as follows.

\[
z = \left\lfloor \frac{(GC(T) - GC(t_{SAY}))}{\nu s} \right\rfloor + 1
\]

**Step 6** \( t_{S1}, t_{S2}, \ldots, t_{Sc} \) are calculated by (18). Each transportation by \( S \)-mode is accomplished such that at these times the amount of inventory increases by \( \nu s \), except the last one. The last transportation amount is given as the remaining part of \( GC(t) \).

### 6 Some Considerations on the Efficiency of \( SA \)-Mode

The above obtained equation (12) is a very special case of the expected profit obtained by allocating the three modes of transportation. On the basis of this consideration several much more realistic situation should be considered. For example, the total consuming function may differ much depending on products and advertisement.

However, several interesting features of the sales become apparent. First, the efficiency of the introduction of \( SA \)-mode is apparent from the presented figures, because, before \( t_s \), \( S \)-mode transportation cannot be applied and yet the consumption increases considerably. Applying \( A \)-mode at this stage usually costs much. At this stage the speed of consumption...
increases much as is shown in the figure of speed of consumption. Therefore, lower cost and larger amount of transportation is most efficient.

The value-cost curve shows there exist a peak at the middle transportation time. Also from this figure, the transportation of medium speed, in the present case SA-mode, is most efficient. The difference of consuming speed curve caused by the speed parameter $\omega$ indicates that the obtained profit increases as $\omega$ increases. This means, speeding up the sales, by means of advertisement or so, is efficient in increasing the total profit.

7 Conclusion

The importance and efficiency of the third method of intercontinental transportation of products was shown here. This method is called sea-and-air because it uses these two methods. Most popular sea-and-air transportation is performed through Japan. It is rather a favorable situation for Japan both for sea and air transportation. These situations will be shown in another report. A theoretical analysis for the optimal allocation of these three methods was first done in this research. Although the condition of this report situation of the research is very primitive, it was found that the some typical phenomena can be explained. At the same time some useful strategy applying this method was shown. The nonlinear functions utilized here, which are estimated through theoretical considerations, might be much more complicated in reality and might be different for each kind of product. They should be constructed and polished for each product. Although it seems a fairly difficult task, it might have to be done for/by each product in the future.
References


